Section 5.5

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(121) Combine as a single logarithm and, if possible, simplify.

(a) $\frac{1}{2}\ln x + 3\ln y - 2\ln z$ (b) $\log(x^2-9) - \log(x-3)$

 # 121

Part-a

$$\frac{1}{2}\ln x + 3\ln y - 2\ln z$$

$$= \ln x^{1/2} + \ln y^3 - \ln z^2$$

$$= \ln\left(\frac{x^{1/2}\,y^3}{z^2}\right)$$

Part-b

$$\log(x^2-9) - \log(x-3)$$

$$= \log\left(\frac{x^2-9}{x-3}\right)$$

$$= \log\left[\frac{(x-3)(x+3)}{x-3}\right]$$

$$= \log(x+3)$$

 # 122

(122) Express in terms of sums and differences of logarithms.

(a) $\log\left(\frac{x^3y^9}{z}\right)^{2/3}$ (b) $\ln\frac{\sqrt{x+1}}{(x-3)^2}$

Part-a

$$\log\left(\frac{x^3y^9}{z}\right)^{2/3} = \frac{2}{3}\log\left(\frac{x^3y^9}{z}\right)$$

$$= \frac{2}{3}(\log x^3 + \log y^9 - \log z)$$

$$= \frac{2}{3}(3\log x + 9\log y - \log z)$$

$$= 2\log x + 6\log y - \frac{2}{3}\log z$$

Part-b

$$\ln\frac{\sqrt{x+1}}{(x-3)^2} = \ln\sqrt{x+1} - \ln(x-3)^2$$

$$= \frac{1}{2}\ln(x+1) - 2\ln(x-3)$$

 # 123

(123) The function describing loudness is $D(x) = 160 + 10\log x$, where x is the intensity of the sound (in watts/m^2) and $D(x)$ is the loudness (in decibels). According to the National Institute of Deafness and Other Communication Disorders, prolonged exposure to sounds of 85 dB or more can cause damage to your hearing. What is the intensity of the noise? Round your answer to 8 decimal places.

We want to find x so that D(x) = 85

$160 + 10\log x = 85$

Subtract 160 from each side

$10\log x = -75$

Divide both sides by 10

$\log x = -7.5$

$x = 10^{-7.5}$

$x \approx$ **0.00000003** (watts/ m^2)

Section 5.6

Section 5.6

In Exercises 124-127, solve the given equation algebraically.

(124) $3^{2x+1} = 3^{5-x}$

(125) $3^{2x+1} = e^x$

(126) $\ln(3-2x) = 2\ln x$

(127) $\log_9 2x + \log_9\left(x-\frac{5}{2}\right) = \frac{1}{2}$

 # 124

$3^{2x+1} = 3^{5-x}$

The bases are equal so we equate the exponents

$2x + 1 = 5 - x$

$2x + x = 5 - 1$

$3x = 4$

Divide both sides by 3

$x = \frac{4}{3}$

 # 125

$3^{2x+1} = e^{x}$

Get the natural logarithm of both sides

$\ln(3^{2x+1}) = \ln(e^{x})$

$(2x + 1)\ln 3 = x \ln e$

$(2x + 1)\ln 3 = x$ because $\ln e = 1$

$(2 \ln 3)x + \ln 3 = x$

$(2 \ln 3)x - x = -\ln 3$

$[(2 \ln 3) - 1]x = -\ln 3$

$x = \frac{-ln\ 3}{(2 \ln 3) - 1}$

$x = \frac{ln\ 3}{1-2 \ln 3}$ which is approximately -0.91763259

 # 126

$\ln(3 - 2x) = 2 \ln x$

$\ln(3 - 2x) = \ln x^2$

$3 - 2x = x^2$

$x^2 + 2x - 3 = 0$

$(x + 3)(x - 1) = 0$

$x = -3$ or $x = 1$

Disregard $x = -3$ because the logarithm of a negative number is not real

$x = 1$

 # 127

$\log_9 2x + \log_9\left(x - \frac{5}{2}\right) = \frac{1}{2}$

$\log_9 2x\left(x - \frac{5}{2}\right) = \frac{1}{2}$

$2x\left(x - \frac{5}{2}\right) = 9^{1/2}$ [Exponential form]

$2x^2 - 5x = 3$ because $9^{1/2} = 3$

$2x^2 - 5x - 3 = 0$

$(2x + 1)(x - 3) = 0$

$x = \frac{-1}{2}$ or $x = 3$

Disregard $x = \frac{-1}{2}$ because the logarithm of a negative number is not real

$x = 3$

 # 128

(128) Solve the following reducible quadratic equations. Express answers in exact form.

(a) $e^{2x} - 7e^x + 10 = 0$ (b) $\ln^2 x + 2\ln x - 8 = 0$

Part-a

Let $y = e^x$

$y^2 - 7y + 10 = 0$

$(y - 2)(y - 5) = 0$

$y = 2$ or $y = 5$

$e^x = 2$ or $e^x = 5$

$x = \ln 2$ or $x = \ln 5$

Part-b

Let $y = \ln x$

$y^2 + 2y - 8 = 0$

$(y + 4)(y - 2) = 0$

$y = -4$ or $y = 2$

$\ln x = -4$ or $\ln x = 2$

$x = e^{-4}$ or $x = e^2$

$x = \frac{1}{e^4}$ or $x = e^2$

 # 129

(129) $10,000 is invested at an interest rate of 7.25% per year, compounded continuously.
(a) Find the doubling time.
(b) How long will it take for the investment to grow to $12,000?

Part-a

We want to solve for t so that

$$10000e^{0.0725\,t} = 20000$$

Divide both sides by 10000

$$e^{0.0725\,t} = 2$$

$$0.0725\,t = \ln 2$$

Divide both sides by 0.0725

$$t = \frac{\ln 2}{0.0725} \approx \textbf{9.56 years}$$

Part-b

We want to solve for t so that

$$10000e^{0.0725\,t} = 12000$$

Divide both sides by 10000

$$e^{0.0725\,t} = 1.2$$

$$0.0725\,t = \ln 1.2$$

Divide both sides by 0.0725

$$t = \frac{\ln 1.2}{0.0725} \approx \textbf{2.51478 years}$$

#130

(130) The percentage P of radioactive carbon-14 remaining in a fossil after t years is $P(t) = 100\left(\frac{1}{2}\right)^{t/5700}$. Suppose scientists discover bone fragments that had lost 80% of their carbon-14. How old were the bone fragments at the time of the discovery?
(Hint: Remaining carbon-14 is 100% - 80% = 20%.)

We want to solve for t so that $P(t) = 20$

$$100\left(\frac{1}{2}\right)^{t/5700} = 20$$

Divide both sides by 100

$$\left(\frac{1}{2}\right)^{t/5700} = 0.20$$

Get the logarithm of both sides

$$\log\left(\frac{1}{2}\right)^{t/5700} = \log 0.20$$

$$\frac{t}{5700}\log\left(\frac{1}{2}\right) = \log 0.20$$

$$t = \frac{5700 \log 0.20}{\log\left(\frac{1}{2}\right)} \approx \textbf{13234.99 years}$$